Conformal Manifolds and Chiral Rings

from 1602.05971, sec. 1.

Conformal manifolds

CFT_d , $\mathcal{O}$ = scalar marginal operator.

Deformation $\delta S = \lambda \int d^dx\, \mathcal{O} \implies \beta = \frac{d\lambda}{d\log\mu} = \underbrace{\beta_1 \lambda^2 + \beta_2 \lambda^3 + \dots}_{\text{pert. theory}}$

If all $\beta_i = 0$, then $\mathcal{O}$ = exactly marginal and λ defines a line of CFTs, along which critical exponents can vary continuously.

If several such operators $\longrightarrow$ "conformal manifold"

Zamolodchikov metric $\langle \mathcal{O}_i(x)\, \mathcal{O}_j(0) \rangle_{\{\lambda\}} = \frac{g_{ij}(\lambda)}{x^{2d}}$ $\lambda = (\lambda^1, \dots, \lambda^n)$

When do all the β_i vanish?

- Common in $c=1$ models in 2d
- CFT with current algebra symmetry
- CFT with susy $(2 \leq d \leq 4)$

Example : 4d $\mathcal{N}=2$.

$\boxed{\begin{array}{l} * \text{ Kähler conformal manifold, with coords } (\tau^i, \bar{\tau}^i) \text{ descendants of } \mathcal{N}=2 \text{ chiral primaries of dim } 2 \\ * \text{ Every marginal is exactly marginal} \\ * \text{ Trivial Kähler class } (\Rightarrow \text{conf manifold non-compact}) \end{array}}$

Chiral ring of $\mathcal{N}=2$ SCFTs

SCA $\to$
- Conf algebra $so(5,1) \rightsquigarrow (\Delta, j_L, j_R)$
- Poincaré supercharges $Q^a_\alpha, \bar{Q}^a_{\dot\alpha}$ and superconf $S^\alpha_a, \bar{S}^{\dot\alpha}_a$
- $su(2)_R \times u(1)_R$ R-sym. $\rightsquigarrow (s, R)$

Define : • "chiral primary" = superconformal primary annihilated by $\bar{Q}^{a}_{\dot{\alpha}}$.

$$\Longrightarrow \quad \Delta = \frac{R}{2} \qquad j_r = s = 0$$

(and in all known examples $j_\ell = 0$ as well.)

↳ They form the chiral ring which:

* is freely generated
* # generators = dim (Coulomb branch)

• "operators $\mathcal{H}_I$" = superconformal primary annihilated by Q^1_α and $\bar{Q}^1_{\dot{\alpha}}$

$$\Longrightarrow \quad \Delta = 2s \qquad j_\ell = j_r = R = 0$$

* They also form a ring
* ring <u>not</u> freely generated
* operators in this ring parametrize the Higgs branch.